Topic 9
Trigonometric Ratios

Bronze, Silver, Gold
Worksheets for
AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 25

Q1

Find the exact value of tan 30° × sin 60°
Give your answer in its simplest form.

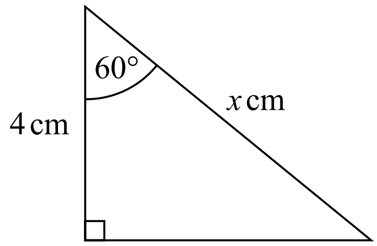
(Total for Question 1 is 2 marks)

Q2

(a) Write down the exact value of tan 45°

(1)

Here is a right-angled triangle.



cos 60° = 0.5

(b) Work out the value of x.

(2)

(Total for Question 2 is 3 marks)

Q3

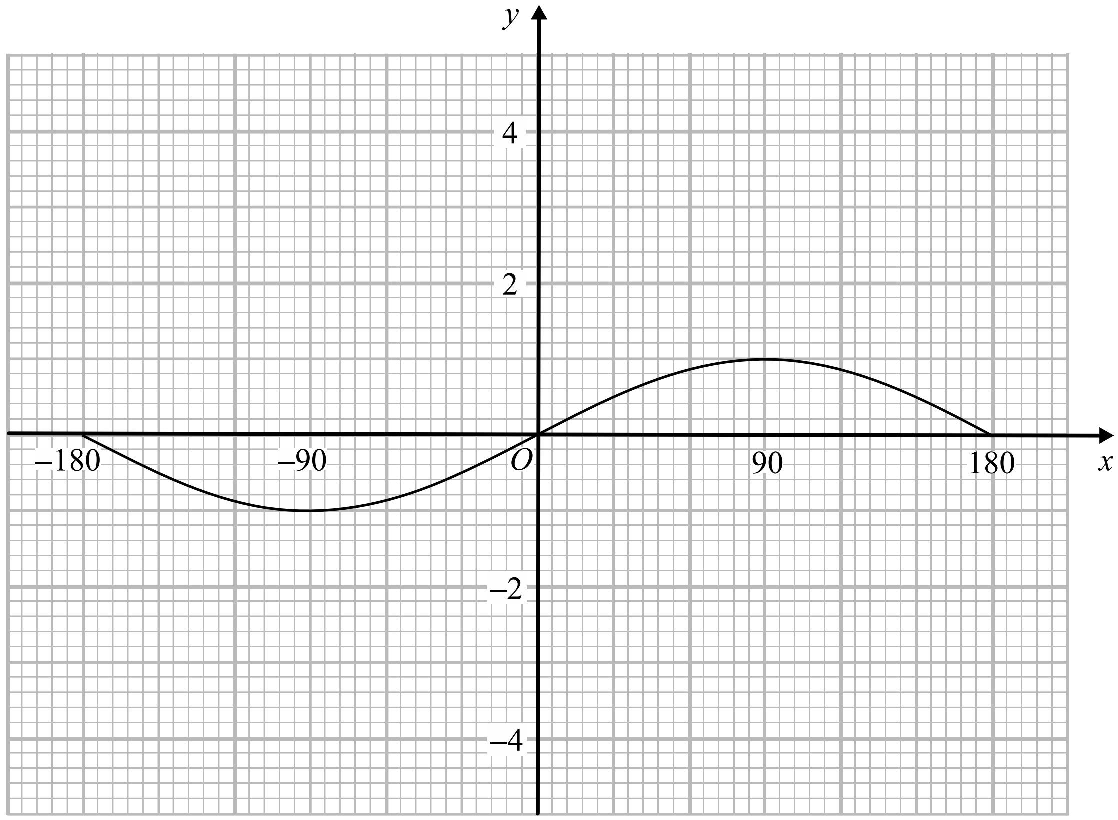
In the triangle ABC, $AB = 1$m, $AC = \sqrt{3}$m, angle $ABC = 60°$ and angle $BCA = x°$

Find the two possible values for x.

(Total for Question 3 is 4 marks)

Q4

Here is the graph of $y = \sin x°$ for $-180 \leqslant x \leqslant 180$



On the grid, sketch the graph of $y = \sin x° - 2$ for $-180 \leqslant x \leqslant 180$

(Total for Question 4 is 2 marks)

Q5

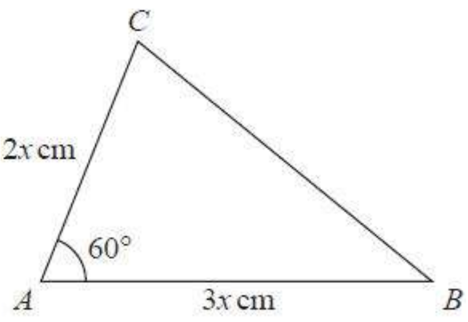


Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle ABC is $18\sqrt{3}$ cm^2

(a) Show that $x = 2\sqrt{3}$

(3)

(b) Hence find the exact length of BC, giving your answer as a simplified surd.

(3)

(Total for Question 5 is 6 marks)

Q6

In triangle RPQ,

$RP = \sqrt{3}$ cm
$PQ = 1$ cm
Angle $PRQ = 30°$

(a) Assuming that angle PQR is an acute angle,
calculate the area of triangle RPQ.
Give your answer in exact form.

(4)

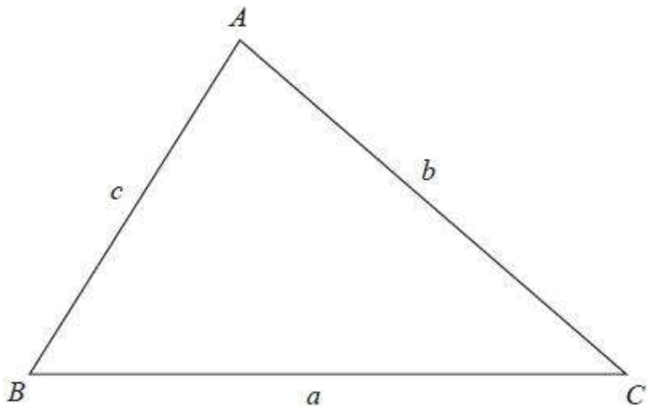
(b) If you did not know that angle PQR is an acute angle, what effect would this have on your calculation of the area of triangle RPQ?

(1)

(Total for Question 6 is 5 marks)

Q7

The diagram shows an acute-angled triangle *ABC*.



Prove that area of triangle $ABC = \frac{1}{2}ab\sin C$

(Total for Question 7 is 3 marks)

Bronze Mark Scheme

Q1.

	$\frac{1}{2}$	M1	for $\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$ **or** $\frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2}$ **or** $(\frac{1}{2} \div \frac{\sqrt{3}}{2}) \times \frac{\sqrt{3}}{2}$ **OR** $\tan 30 = \frac{1}{\sqrt{3}}$ oe **or** $\sin 60 = \frac{\sqrt{3}}{2}$
		A1	for $\frac{1}{2}$ or 0.5

Q2.

(a)	1	B1	cao	
(b)	8	M1	starts process, eg $\cos(60) = \frac{4}{x}$ or $0.5 = \frac{4}{x}$ oe or $\sin 30 = \frac{4}{x}$ or $\frac{\sin 30}{4} = \frac{\sin 90}{x}$ oe	All three elements of cos, 4, x must be present in an equation. eg cos = $4/x$ is acceptable but $\cos(4/x)$ is insufficient
		A1	cao	

Q3.

Question Number	Scheme	Marks
	$\frac{\sin x}{1} = \frac{\sin 60°}{\sqrt{3}}$	M1
	$(\sin x) = \frac{1 \times \sin 60°}{\sqrt{3}} \left(= \frac{1}{2}\right)$	A1
	$x =$ awrt 30 **and** 150	dM1 A1 **(4)** **[4]**

Q4.

18	Graph drawn	C2 (C1	for graph translated by −2 in the y direction for a graph translated in the y direction **OR** for a correct graph through four of the five key points)	Key points: (–180, –2), (–90, –3), (0, –2), (90, –1), (180, –2)

Q5.

Question	Scheme	Marks	AOs
(a)	Uses $18\sqrt{3}=\frac{1}{2}\times 2x\times 3x\times\sin 60°$	M1	1.1a
	Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $x^2=k$ oe	M1	1.1b
	$x=\sqrt{12}=2\sqrt{3}$ *	A1*	2.1
		(3)	
(b)	Uses $BC^2=\left(6\sqrt{3}\right)^2+\left(4\sqrt{3}\right)^2-2\times 6\sqrt{3}\times 4\sqrt{3}\times\cos 60°$	M1	1.1b
	$BC^2=84$	A1	1.1b
	$BC=2\sqrt{21}$ (cm)	A1	1.1b
		(3)	
			(6 marks)

Notes
(a) **M1:** Attempts to use the formula $A=\frac{1}{2}ab\sin C$. If the candidate writes $18\sqrt{3}=\frac{1}{2}\times 5x\times \sin 60°$ **without** sight of a previous correct line then this would be M0 **M1:** Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ or awrt 0.866 and proceeds to $x^2=k$ oe such as $px^2=q$ This may be awarded from the correct formula or $A=ab\sin C$ **A1*:** Look for $x^2=12\Rightarrow x=2\sqrt{3}$, $x^2=4\times 3\Rightarrow x=2\sqrt{3}$ or $x=\sqrt{12}=2\sqrt{3}$ This is a given answer and all aspects must be correct including one of the above intermediate lines. It cannot be scored by using decimal equivalents to $\sqrt{3}$ Alternative using the given answer of $x=2\sqrt{3}$ **M1:** Attempts to use the formula $A=\frac{1}{2}\times 4\sqrt{3}\times 6\sqrt{3}\sin 60°$ oe **M1:** Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $A=18\sqrt{3}$ **A1*:** Concludes that $x=2\sqrt{3}$ **(b)** **M1:** Attempts the cosine rule with the sides in the correct position. This can be scored from $BC^2=(3x)^2+(2x)^2-2\times 3x\times 2x\times\cos 60°$ as long as there is some attempt to substitute x in later. Condone slips on the squaring **A1:** $BC^2=84$ Accept $BC^2=7\times 12$, $BC=\sqrt{84}$ or $BC=2\sqrt{21}$ If they replace the surds with decimals they can score the A1 for $BC^2=$ awrt 84.0 **A1:** $BC=2\sqrt{21}$ Condone other variables, say $x=2\sqrt{21}$, but it cannot be scored via decimals.

Q6.

Paper 1MA1: 2H			
Question	**Working**	**Answer**	**Notes**
(a)		$\frac{\sqrt{3}}{2}$	P1 start to process eg draw a labelled triangle or use of sine rule $\frac{\sin Q}{\sqrt{3}} = \frac{\sin 30}{1.0}$ P1 process to find of Q eg. $Q = \sin^{-1}\left[\frac{\sin 3Q}{1.0} \times \sqrt{3}\right]$ P1 process to find area of triangle PRQ. A1 $\frac{\sqrt{3}}{2}$
(b)			C1 angle PRQ is obtuse so need to find area of two triangles.

Q7.

Question	Working	Answer	Mark	Notes
		Shown	M1	for use of sine to find height, e.g. $\sin C = \frac{h}{b}$
			M1	for use of expression for the height of the triangle, e.g. area $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2}ab\sin C$
			C1	for complete proof

Silver Questions

Calculators may not be used

The total mark for this section is 29

Q1

In the triangle *ABC*, $AB = 5\sqrt{6}$ cm, $AC = 4$ cm, angle $ABC = 45°$ and angle $BCA = x°$

Find the two possible values for x, giving your answers in exact form.

(Total for Question 1 is 4 marks)

Q2

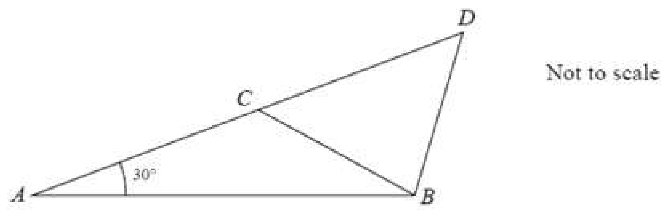


Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, *AB*, *BD*, *BC* and *AD*.

Given $AB = \sqrt{2}$m, $BC = BD = 1$m and angle $BAC = 30°$

Find, the size of angle *ACB*.

(Total for Question 2 is 3 marks)

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Q3

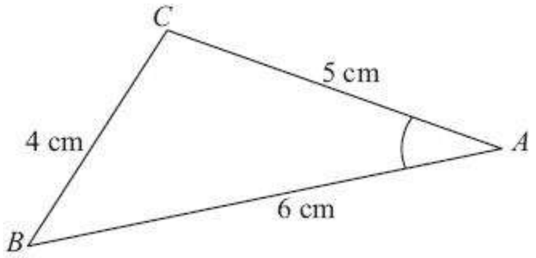


Figure 1

Figure 1 shows the triangle ABC, with $AB = 6$ cm, $BC = 4$ cm and $CA = 5$ cm.

(a) Show that $\cos A = \frac{3}{4}$.

(3)

(b) Hence, or otherwise, find the exact value of $\sin A$.

(2)

(Total for Question 3 is 5 marks)

Q4

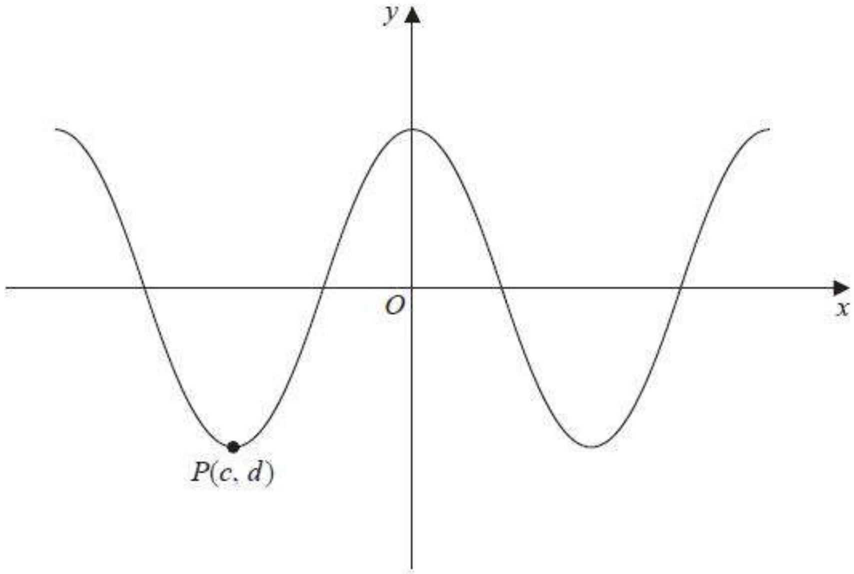


Figure 3

Figure 3 shows part of the curve with equation $y = 3 \cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which
a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3 \cos x°$ to the curve with equation

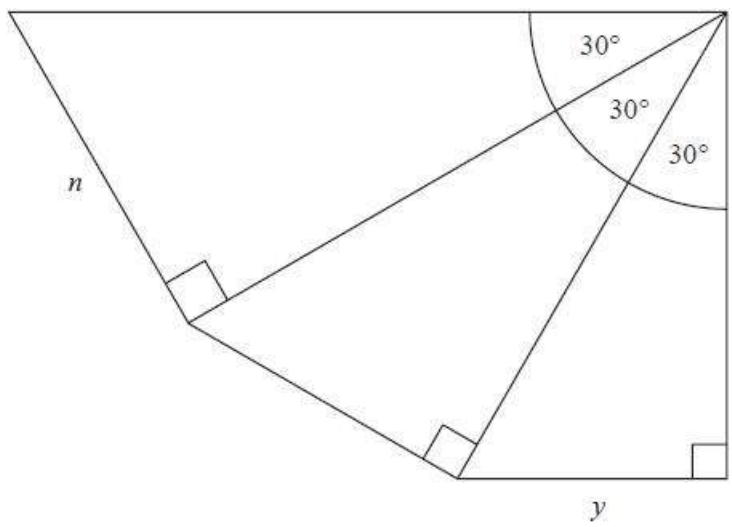
(i) $y = 3\cos\left(\frac{x°}{4}\right)$

(ii) $y = 3 \cos (x - 36)°$

(2)

(Total for Question 4 is 3 marks)

Q5



The diagram shows three right-angled triangles.

Prove that $y = \frac{3}{4}n$

(Total for Question 5 is 4 marks)

Q6

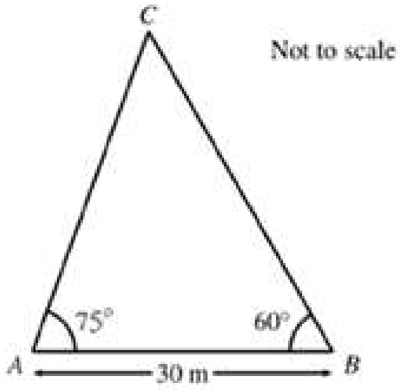


Figure 1

A triangular lawn is modelled by the triangle *ABC*, shown in Figure 1. The length *AB* is to be 30 m long.

Given that angle *BAC* = 75° and angle *ABC* = 60°,

(a) Calculate the length AC **(2)**

Given that BC=15+15$\sqrt{3}$

(b) Calculate the area of the lawn in exact form.

(2)

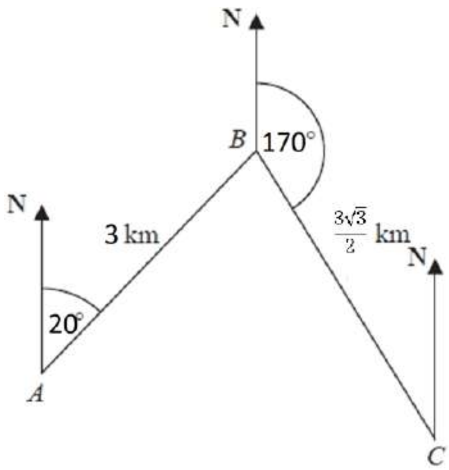
(c) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

(Total for Question 6 is 5 marks)

Q7

The diagram shows the positions of three towns, Acton (A), Barston (B) and Chorlton (C).



Barston is 3 km from Acton on a bearing of 020°

Chorlton is $\dfrac{3\sqrt{3}}{2}$ km from Barston on a bearing of 170°

Find the bearing of Chorlton from Acton.
You must show all your working.

(Total for Question 7 is 5 marks)

Silver Mark Scheme

Q1.

Question Number	Scheme	Marks
	$\frac{\sin x}{5\sqrt{6}} = \frac{\sin 45}{4}$	M1
	$(\sin x) = \frac{5\sqrt{6} \times \sin 45}{4}$	A1
	$x = 60$ and 120	dM1 A1
		(4) **[4]**

Q2.

Question	Scheme	Marks	AOs
	States $\frac{\sin\theta}{\sqrt{2}} = \frac{\sin 30°}{1}$	M1	1.1b
	Finds $\theta =$ awrt $45°$ or awrt $135°$	A1	1.1b
	$=$ awrt $135°$	A1	1.1b
		(3)	

Q3.

Question number	Scheme	Marks
	(a) $4^2 = 5^2 + 6^2 - (2\times5\times6\cos\theta)$	M1
	$\cos\theta = \dfrac{5^2+6^2-4^2}{2\times5\times6}$	A1
	$\left(=\dfrac{45}{60}\right)=\dfrac{3}{4}$ (*)	A1cso (3)
	(b) $\sin^2 A + \left(\dfrac{3}{4}\right)^2 = 1$ (or equiv. Pythag. method)	M1
	$\left(\sin^2 A = \dfrac{7}{16}\right)$ $\sin A = \dfrac{1}{4}\sqrt{7}$ or equivalent exact form, e.g. $\sqrt{\dfrac{7}{16}}$, $\sqrt{0.4375}$	A1 (2)
		5
	(a) M: Is also scored for $5^2 = 4^2 + 6^2 - (2\times4\times6\cos\theta)$ or $6^2 = 5^2 + 4^2 - (2\times5\times4\cos\theta)$ or $\cos\theta = \dfrac{4^2+6^2-5^2}{2\times4\times6}$ or $\cos\theta = \dfrac{5^2+4^2-6^2}{2\times5\times4}$. 1st A: Rearranged correctly and numerically correct (possibly unsimplified), in the form $\cos\theta = ...$ or $60\cos\theta = 45$ (or equiv. in the form $p\cos\theta = q$). <u>Alternative</u> (verification): $4^2 = 5^2 + 6^2 - \left(2\times5\times6\times\dfrac{3}{4}\right)$ [M1] Evaluate correctly, at least to $16 = 25 + 36 - 45$ [A1] Conclusion (perhaps as simple as a tick). [A1cso] (Just achieving $16 = 16$ is insufficient without at least a tick). (b) M: Using a correct method to find an equation in $\sin^2 A$ or $\sin A$ which would give an exact value. <u>Correct answer without working</u> (or with unclear working or decimals): Still scores both marks.	

Pearson

Q4.

Question	Scheme	Marks	AOs
(a)	$(-180°,-3)$	B1	1.1b
		(1)	
(b)	(i) $(-720°,-3)$	B1ft	2.2a
	(ii) $(-144°,-3)$	B1 ft	2.2a
		(2)	
			(3 marks)

(a)

B1: Deduces that $P(-180°,-3)$ or $c=-180^{(o)}, d=-3$

(b)(i)

B1ft: Deduces that $P'(-720°,-3)$ Follow through on their $(c,d)\to(4c,d)$ where d is negative

(b)(ii)

B1ft: Deduces that $P'(-144°,-3)$ Follow through on their $(c,d)\to(c+36°,d)$ where d is negative

Q5.

Question	Working	Answer	Mark	Notes
		Proof	B1	for using any correct trig value for 30°, e.g. $\sin 30 = 0.5$, $\cos 30 = \dfrac{\sqrt{3}}{2}$ or $\tan 30 = \dfrac{1}{\sqrt{3}}$
			M1	for hypotenuse of small triangle = $2y$ or hypotenuse of large triangle = $2n$
			A1	for method to find the hypotenuse of middle triangle, e.g. $\sqrt{(2n)^2-n^2}$ $(=\sqrt{3}n)$
			A1	for a correct equation linking y and n and correct working leading to the given result

Q6.

Question	Scheme		Marks	AOs
(a)	Finds third angle of triangle and uses or states $\dfrac{x}{\sin 60°} = \dfrac{30}{\sin 45°}$		M1	2.1
	So $x = \dfrac{30\sin 60°}{\sin 45°}$ $(= 15\sqrt{6})$		A1	1.1b
(b)	Area $= \dfrac{1}{2} \times 30 \times (15 + 15\sqrt{3}) \times \sin 60$		M1	3.1a
	$= 675 + 225\sqrt{3}$ m²		A1ft	1.1b
			(4)	
(c)	Plausible reason e.g. Because the angles and the side length are not given to four significant figures Or e.g. The lawn may not be flat		B1	3.2b
			(1)	
				(5 marks)

Q7.

Question	Answer	Mark	Mark scheme	Additional guidance
	80	P1	for using bearings to determine *ABC* as 30°	Accept 67 written on the diagram.
		P1	for using the cosine rule to find *AC* eg $(AC^2=)\ 3^2 + (\frac{3\sqrt{3}}{2})^2 - 2 \times 3 \times \frac{3\sqrt{3}}{2} \cos(30)$ $AC=\frac{3}{2}$	Accept correct substitution into RHS of equation Accept *AC* in the range 9.41 to 9.42
		P1	(dep P1) for using the sine rule to find angle *BAC* $\frac{\sin(30)}{\frac{3}{2}} = \frac{\sin(BAC)}{\frac{3\sqrt{3}}{2}}$	
		P1	for rearranging $\frac{\sin(30)}{\frac{3}{2}} \times \frac{3\sqrt{3}}{2} = \sin(BAC)$ for angle *BAC* = 60	Accept any equivalent form with values substituted
		A1	for angle 080°	If the correct answer is given without supportive evidence award 0 marks. Condone missing "0" at the front. If an answer within the range is seen in working and rounded incorrectly award full marks.

Gold Questions

Calculators may not be used

The total mark for this section is 29

Q1

Figure 1

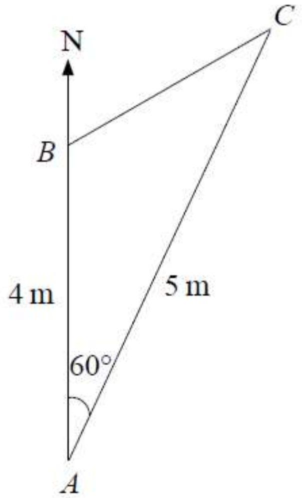


Figure 1 shows 3 yachts A, B and C which are assumed to be in the same horizontal plane. Yacht B is 4 m due north of yacht A and yacht C is 5 m from A. The bearing of C from A is 060°.

Calculate the distance between yacht B and yacht C, in exact form.

(Total for Question 1 is 3 marks)

Q2

In a triangle ABC, side AB has length 10 cm, side AC has length 5 cm, and angle $BAC = \theta$ where θ is measured in degrees. The area of triangle ABC is 15cm^2

(a) Find the two possible values of $\cos\theta$

(4)

Given that BC is the longest side of the triangle,

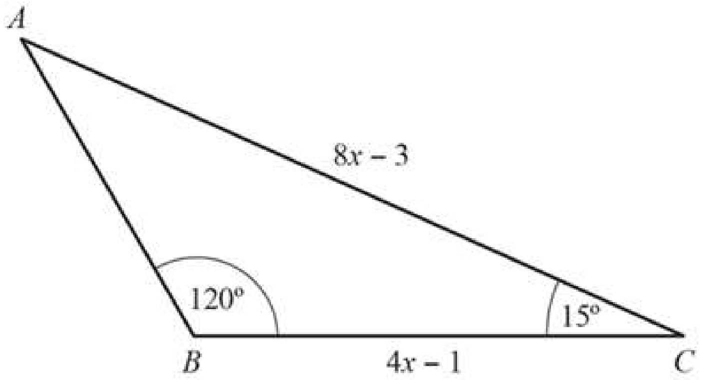
(b) find the exact length of BC.

(2)

(Total for Question 2 is 6 marks)

Q3

The diagram shows $\triangle ABC$ with $AC = 8x - 3$, $BC = 4x - 1$, $\angle ABC = 120°$ and $\angle ACB = 15°$.



(*a*) Show that the exact value of x is $\dfrac{9+\sqrt{6}}{20}$.

(7)

(*b*) Find the area of $\triangle ABC$, giving your answer in exact form

(3)

(Total for Question 3 is 10 marks)

Q4

A buoy is a device which floats on the surface of the sea and moves up and down as waves pass.

For a certain buoy, its height, above its position in still water, y in metres, is modelled by a sine function of the form $y = \dfrac{1}{2}\sin 180t°$, where t is the time in seconds.

(*a*) Sketch a graph showing the height of the buoy above its still water level for $0 \leqslant t \leqslant 10$ showing the coordinates of points of intersection with the t-axis.

(3)

(*b*) Write down the number of times the buoy is 0.4 m above its still water position during the first 10 seconds.

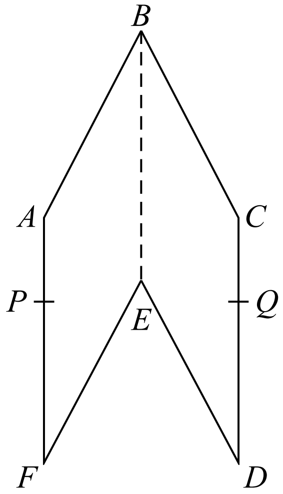
(1)

(*c*) Give one reason why this model might not be realistic.

(1)

(Total for Question 4 is 5 marks)

Q5 The diagram shows a hexagon *ABCDEF*.



ABEF and *CBED* are congruent parallelograms where $AB = BC = x$ cm.
P is the point on *AF* and *Q* is the point on *CD* such that $BP = BQ = 10$ cm.

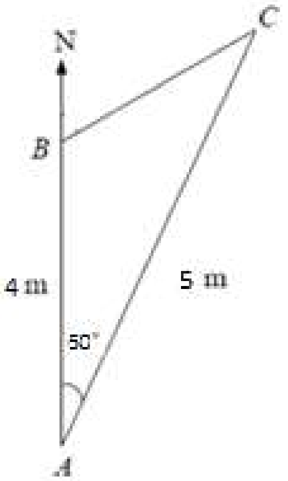
Given that angle $ABC = 30°$,

prove that $\cos PBQ = 1 - \frac{(2-\sqrt{3})}{200}x^2$

(Total for Question 5 is 5 marks)

Gold Mark Scheme

Q1.

Question Number	Scheme	Marks
	Figure 1  $BC^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \cos 60^\circ$ $BC = 4\text{m}$	M1 A1 A1 (3) [3]

Q2.

Question	Scheme	Marks	AOs
(a)	Uses $15=\frac{1}{2}\times5\times10\times\sin\theta$	M1	1.1b
	$\sin\theta=\frac{3}{5}$ oe	A1	1.1b
	Uses $\cos^2\theta=1-\sin^2\theta$	M1	2.1
	$\cos\theta=\pm\frac{4}{5}$	A1	1.1b
		(4)	
(b)	Uses $BC^2=10^2+5^2-2\times10\times5\times"-\frac{4}{5}"$	M1	3.1a
	$BC=\sqrt{205}$	A1	1.1b
		(2)	
		(6 marks)	

Notes

(a)

M1: Uses the formula Area $=\frac{1}{2}ab\sin C$ in an attempt to find the value of $\sin\theta$ or θ

A1: $\sin\theta=\frac{3}{5}$ oe This may be implied by $\theta=$ awrt $36.9°$ or awrt 0.644 (radians)

M1: Uses their value of $\sin\theta$ to find two values of $\cos\theta$ This may be scored via the formula $\cos^2\theta=1-\sin^2\theta$ or by a triangle method. Also allow the use of a graphical calculator or candidates may just write down the **two values**. The values must be symmetrical $\pm k$

A1: $\cos\theta=\pm\frac{4}{5}$ or ±0.8 Condone these values appearing from ±0.79....

(b)

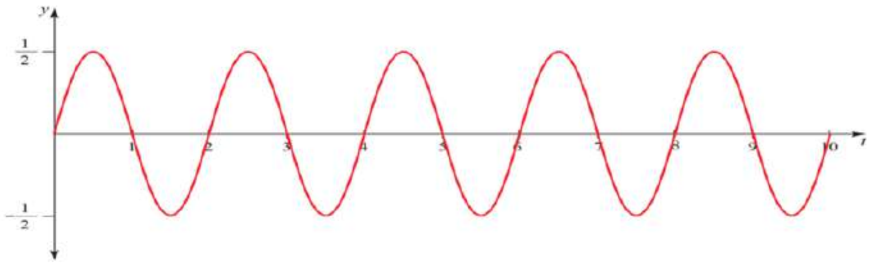
M1: Uses a suitable method of finding the longest side. For example chooses the negative value (or the obtuse angle) and proceeds to find BC using the cosine rule. Alternatively works out BC using both values and chooses the larger value. If stated the cosine rule should be correct (with a minus sign). Note if the sign is +ve and the acute angle is chosen the correct value will be seen. It is however M0 A0

A1: $BC=\sqrt{205}$

Q3.

3a	$\angle A = 45^\circ$ seen or implied in later working.	**B1**
	Makes an attempt to use the sine rule, for example, writing $\dfrac{\sin 120^\circ}{8x-3} = \dfrac{\sin 45^\circ}{4x-1}$	**M1**
	States or implies that $\sin 120^\circ = \dfrac{\sqrt{3}}{2}$ and $\sin 45^\circ = \dfrac{\sqrt{2}}{2}$ **NOTE:** Award ft marks for correct work following incorrect values for sin 120° and sin 45°	**A1**
	Makes an attempt to solve the equation for x. Possible steps could include: $\dfrac{\sqrt{3}}{16x-6} = \dfrac{\sqrt{2}}{8x-2}$ or $\dfrac{\sqrt{6}}{16x-6} = \dfrac{1}{4x-1}$ or $\dfrac{3}{16x-6} = \dfrac{\sqrt{6}}{8x-2}$ $(8\sqrt{3})x - 2\sqrt{3} = (16\sqrt{2})x - 6\sqrt{2}$ or $(4\sqrt{6})x - \sqrt{6} = 16x - 6$ or $24x - 6 = (16\sqrt{6})x - 6\sqrt{6}$ $6\sqrt{2} - 2\sqrt{3} = x(16\sqrt{2} - 8\sqrt{3})$ or $(4\sqrt{6})x - \sqrt{6} = 16x - 6$ or $12x - 3 = (8\sqrt{6})x - 3\sqrt{6}$	**M1ft**
	$x = \dfrac{6\sqrt{2} - 2\sqrt{3}}{16\sqrt{2} - 8\sqrt{3}}$ or $x = \dfrac{6-\sqrt{6}}{16 - 4\sqrt{6}}$ or $x = \dfrac{3\sqrt{6} - 3}{8\sqrt{6} - 12}$ o.e.	**A1ft**
	Makes an attempt to rationalise the denominator by multiplying top and bottom by the conjugate. Possible steps could include: $x = \dfrac{(3\sqrt{2} - \sqrt{3})}{(8\sqrt{2} - 4\sqrt{3})} \times \dfrac{(8\sqrt{2} + 4\sqrt{3})}{(8\sqrt{2} + 4\sqrt{3})}$ $\quad x = \dfrac{48 + 12\sqrt{6} - 8\sqrt{6} - 12}{128 - 48}$ $\quad x = \dfrac{36 + 4\sqrt{6}}{80}$	**M1ft**
	States the fully correct simplifed version for x. $\quad x = \dfrac{9+\sqrt{6}}{20}$ *	**A1***
	NOTE: Award ft marks for correct work following incorrect values for sin 120° and sin 45°	**(7 marks)**
3b	States or implies that the formula for the area of a triangle is $\frac{1}{2}ab\sin C$ or $\frac{1}{2}ac\sin B$ or $\frac{1}{2}bc\sin A$	**M1**
	$\dfrac{1}{2}\left(4\left(\dfrac{9+\sqrt{6}}{20}\right)-1\right)\left(8\left(\dfrac{9+\sqrt{6}}{20}\right)-3\right)(\sin 15\ )$ or $\dfrac{1}{2}\quad(\sin 15\ )$.	**M1**
	Finds the correct area is $\dfrac{1}{200}(24 + 11\sqrt{6})(\sqrt{6} - \sqrt{2})$.	**A1**
		(3 marks) **Total** **10 marks**

Q4.

4a		
	Correct shape of sine curve through (0, 0).	**B1**
	Sine curve has max value of $\frac{1}{2}$ and min value of $-\frac{1}{2}$	**B1**
	Sine curve has a period of 2 (can be implied by 5 complete cycles) and passes through (1,0), (2,0),..., (10,0).	**B1**
		(3 marks)
4b	Student states that the buoy will be 0.4 m above the still water level 10 times.	**B1**
		(1 mark)
4c	Sensible and correct reason. For example: A buoy would not move up and down at exactly the same rate during each oscillation. The period of oscillation is likely to change each oscillation. The maximum (or minimum) height is likely to change with time. Waves in the sea are not uniform.	**B1**
	Award the mark for a different explanation that is mathematically correct. For example, stating that the buoy would not move exactly vertically each time.	**(1 mark)**
		total 5 marks

Q5.

		Proof	B1	(indep) for stating $\cos 30 = \frac{\sqrt{3}}{2}$
			M1	for $PQ^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos PBQ$ **or** $AC^2 = x^2 + x^2 - 2 \times x \times x \times \cos 30$ $(=x^2(2-\sqrt{3}))$ oe
			M1	for $\cos PBQ = \frac{10^2 + 10^2 - PQ^2}{2 \times 10 \times 10}$ (implies previous M1)
	cos PBQ= $\frac{10^2 + 10^2 - x^2(2-\sqrt{3})}{200}$ $= \frac{200 - x^2(2-\sqrt{3})}{200}$		M1	for $\cos PBQ = \frac{10^2 + 10^2 - (x^2 + x^2 - 2 \times x \times x \times \cos 30)}{2 \times 10 \times 10}$
			A1	conclusion of proof with all working seen